ırge Temperature and dients*

ᴀɴᴅ E. F. HAMMEL, JR.

sity of California, Los Alamos,
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y have reported measurements of
quid He II flowing through narrow
fferences as large as 1°K. For the
ıces the linear two-fluid equations
erified; integration over the tem-
paper extends the analysis of the
the linear equations are no longer
utions of the Gorter-Mellink non-
ed on the concept of mutual fric-
ed on the assumptions and restric-
grals for heat flow and fountain
ıg a high-speed computer and the
When Vinen's values of the mutual
solutions, the comparison is quite
own that other values of $A(T)$ are
explanation in terms of the vortex
near T_λ. Despite the agreement
nent obtained here, several as yet
low phenomena in small slits; cer-
l, most notably the criteria for the

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the linear equations of motion for
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try, have been reported in two pre-
and II). In interpreting these meas-
linear equations of motion over the
experiments. This approach proved
untain pressure and heat flow over a
than could be accounted for by the

nited States Atomic Energy Commission.
pton, L. I., New York.

linearized nonintegrated theory. However, at sufficiently high heat flows saturation effects appeared producing significant deviations from the predictions of the linear theory.

In this paper we shall discuss the relationship of measurements involving very large heat current densities to solutions of the Gorter-Mellink (*3*) nonlinear thermohydrodynamical equations. The integrated nonlinear equations are found to reduce to the linear equations for small heat flows. For larger heat currents the calculations using Vinen's (*4*) values of $A(T)$ in the mutual friction term are in good quantitative agreement with the observations, except in the neighborhood of the λ-point. However, since the Vinen model of dissipation in He II resulting from vortex line turbulence in the superfluid as applied to the present experimental arrangement predicts the breakdown of the equations near the λ-point, the observed deviations may be considered as qualitative support for the theory.

II. DERIVATION OF INTEGRATED FLOW EQUATIONS

A. Derivation of ∇P and ∇T

In order to obtain solutions to the thermohydrodynamic equations of motion for He II which are applicable to long narrow slits and capillaries, we begin with the following two-fluid equations of motion, including mutual friction[1]:

$$\rho_s \frac{D\mathbf{v}_s}{Dt} = -\left(\frac{\rho_s}{\rho}\right)\nabla P + \rho_s s\nabla T - \mathbf{F}_{sn} \quad (1)$$

$$\rho_n \frac{D\mathbf{v}_n}{Dt} = -\left(\frac{\rho_n}{\rho}\right)\nabla P - \rho_s s\nabla T + \mathbf{F}_{sn} - \eta_n \nabla \times \nabla \times \mathbf{v}_n + (2\eta_n + \eta')\nabla(\nabla\cdot\mathbf{v}_n) \quad (2)$$

where

$$\frac{D\mathbf{v}}{Dt} = \frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)\mathbf{v}.$$

Here the subscripts s and n refer to the superfluid and the normal fluid, η_n is the normal fluid viscosity, and η' is the bulk or second viscosity. The frictional force $\mathbf{F}_{sn}$ accounts for interaction between the superfluid and the normal fluid. The form of this term will be discussed later. Possible other forces acting separately on the normal fluid and on the superfluid are neglected in this treatment.

[1] The equations of motion have been written in various forms, and the correct form for large velocities and including irreversible processes is still controversial. Equations (1) and (2) originate from the ideas of Tisza (*5*), Landau (*6*), London (*7*), and Gorter and Mellink (*3*), and are believed to serve the present purposes well to a good first approximation. The more detailed treatment of the second viscosity terms by Khalatnikov (*8*) is necessary for analyzing experiments on such phenomena as first and second sound; but in experiments on fountain pressure and heat conduction the second viscosity plays a subordinate role and the following more easily handled equations suffice.